

# Phenomenological implications of $S$ -duality symmetry

Ashok Das<sup>a,b</sup> and Jnanadeva Maharana<sup>c,d\*</sup>

<sup>a</sup> *Department of Physics and Astronomy, University of Rochester, Rochester, NY 14627-0171, USA*

<sup>b</sup> *Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Calcutta 700064, India*

<sup>c</sup> *Institute of Physics, Bhubaneswar 751005, India and*

<sup>d</sup> *National Institute of Science Education and Research, Bhubaneswar 751005, India*

(Dated: January 15, 2013)

It is proposed that  $S$ -duality is a fundamental symmetry of nature which is spontaneously broken. Axion and dilaton are identified with the doublet of the  $S$ -duality symmetry group  $SL(2, \mathbb{R})$ . The symmetry is broken at a high scale corresponding to the experimentally estimated axion decay constant  $f_\chi$ . The symmetry breaking mechanism is discussed in analogy with PCAC in pion physics.  $S$ -duality invariant interactions of fermions with axion and dilaton doublet are introduced. The symmetry breaking mechanism contributes negligibly small corrections to fermion masses in the QCD sector. Inspired by universality in string theory, the  $S$ -duality invariant interaction of the axion-dilaton doublet to QCD fermions is proposed to generalize to all fermions. Phenomenological consequences of this broken symmetry are explored.

PACS numbers: 11.30.-j, 11.30.Qc, 14.80.Va

## I. INTRODUCTION

The standard model of particle physics describes the three fundamental forces of nature (excluding gravitation) in the microscopic domain. The electro-weak theory explains the weak as well as the electromagnetic phenomena [1–5] and quantum chromodynamics (QCD) is the accepted theory of strong interactions [6–8]. The standard model has been tested to a great degree of accuracy and there are no serious discrepancies between the theoretical predictions and the experimental results at energies below the weak scale. However, it is believed that the standard model is incomplete in the sense that it contains several arbitrary parameters and does not incorporate gravitational interactions. The grand unified theories (GUTs) were proposed to provide a unified description of the strong and the electro-weak phenomena [9–12]; however, proton decay, which is a crucial prediction of all GUTs, has not been observed as yet. Furthermore, while string theory is a promising candidate to unify all the four fundamental interactions (including gravity), the standard model of particle physics in total is yet to emerge from string theory.

With the running of LHC now, there is great excitement in probing energy regimes beyond the weak scale. This will allow us to observe certain massive particles which are an essential integral part of the standard model and to study their properties. For example, the Higgs boson which is responsible for the spontaneous breakdown of the electro-weak symmetry is expected to be discovered at LHC. Supersymmetry which, among other things, leads to a resolution of the hierarchy problem is expected to be tested at LHC as well. There are, of course, a host of other models which have been proposed as describ-

ing physics beyond the standard model and experiments at LHC will likely decide on their validity. One of the characteristic (and bothersome) features of all endeavors to construct a model beyond the standard model has been the proliferation of particle spectra at the very fundamental level. Guided mainly by symmetry considerations we propose here a model with only one extra particle which may provide a window to access physics beyond the standard model. Some of our predictions can possibly be tested in precision experiments at LHC, underground experiments as well as in cosmological experiments.

To introduce our model, let us recall that strong interactions had a heavy mass puzzle commonly known as the  $U_A(1)$  problem [13]. The resolution of this problem within the context of QCD was shown by 't Hooft [14] to be related to the existence of instanton solutions which essentially change the structure of the QCD vacuum. The instanton solutions effectively lead to an additional term in the QCD Lagrangian density so that we have

$$\mathcal{L} = -\text{Tr} \left( \frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\theta g^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \right), \quad (1)$$

where the field strength tensor is a matrix in the fundamental representation of  $SU(3)$  and we have scaled out the Yang-Mills coupling constant  $g$  from the field strength tensor. The dual of the field strength tensor is defined to be

$$\tilde{F}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\lambda\rho} F_{\lambda\rho}, \quad (2)$$

and  $\theta$  (in (1)) represents the parameter commonly known as the QCD vacuum angle. However, in the complete standard model including the electro-weak theory, the phases of the quark mass matrix  $M$  also contribute to the pseudoscalar density in (1) and this is proportional to  $\text{Arg det } M$ . Therefore, the parameter  $\theta$  in (1) is to be understood as  $\theta_{\text{eff}} = \theta + \text{Arg det } M$ . The  $\theta$  dependent term in (1) is a total divergence and, therefore, does

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\* e-mail: das@pas.rochester.edu, maharana@iopb.res.in

not modify the equations of motion for the gauge fields. However, it violates  $P$  and  $T$  (and, consequently,  $CP$ ) and induces an electric dipole moment for the neutron. The very stringent limits on the neutron electric dipole moment measurements [15] limit the value of the effective  $\theta$  parameter (we continue to write  $\theta$  for simplicity) to  $\theta < 10^{-9} - 10^{-10}$ . Without any natural (theoretical) justification for why (effective)  $\theta$  should be so small, the resolution of the  $U_A(1)$  problem leads to the strong  $CP$  problem.

A possible resolution of the strong  $CP$  problem which is also quite attractive from the cosmological point of view is through the Peccei-Quinn mechanism [16]. Here one promotes the  $\theta$  parameter to a dynamical pseudoscalar field, the axion  $\chi$ , in a way such that the theory has an additional global  $U(1)$  chiral invariance. This symmetry is spontaneously broken (through a choice of the axion potential) so that the axion develops a vacuum expectation value (vev) leading to the  $\theta$  parameter. Being the vev of a field, this can always be adjusted to be small which is the rationale for the solution of the strong  $CP$  problem through the Peccei-Quinn mechanism. The axion is the Nambu-Goldstone boson [17, 18] of the broken  $U(1)$  symmetry introduced by Peccei and Quinn. Although the original model of Peccei-Quinn has been experimentally ruled out, generalizations of the model where the axion is a weakly interacting, long lived and a very light mass particle (also known as the invisible axion) remain quite attractive. We refer the readers to some of the review articles [19–28] for details and for the literature in this subject. The present bounds on the axion parameters are given by

$$m_\chi \simeq 10^{-4} \text{eV}, \quad f_\chi \simeq 10^9 - 10^{12} \text{GeV}, \quad (3)$$

where  $f_\chi$  denotes the axion decay constant (into two gluons). The coupling of the axion to the pseudoscalar density in such models has the generic form

$$\mathcal{L}_\chi = -\frac{\zeta}{f_\chi} \frac{g^2}{16\pi^2} \text{Tr} \left( \chi F_{\mu\nu} \tilde{F}^{\mu\nu} \right), \quad (4)$$

where  $\zeta$  denotes a model dependent constant parameter and the vev of the axion is related to  $\theta$  through a multiplicative factor. A direct search for axions, however, has not been successful thus far. We would like to propose a model based on  $S$ -duality symmetry which can be tested at LHC and if experimental observations validate the symmetry it may lead to an indirect proof of the axion.

Let us note that the Lagrangian density (1) can be rewritten in the form

$$\begin{aligned} \mathcal{L} &= -\frac{1}{16\pi} \text{Im} \left( \text{Tr} \left( \tau (F^{\mu\nu} \pm i \tilde{F}^{\mu\nu}) (F_{\mu\nu} \pm i \tilde{F}_{\mu\nu}) \right) \right) \\ &= -\frac{1}{16\pi} \text{Im} \left( \text{Tr} \left( \pm i \tau F^{\mu\nu} \tilde{F}_{\mu\nu}^\pm \right) \right), \end{aligned} \quad (5)$$

where  $\tau = \pm \frac{\theta g^2}{2\pi} + \frac{4i\pi}{g^2}$  denotes the moduli parameter and  $F_{\mu\nu}^\pm = F_{\mu\nu} \pm i \tilde{F}_{\mu\nu}$ . Note that the angle  $\theta$  has a period

of  $2\pi$  and the resulting equations of motion are invariant under the transformations  $\tau \rightarrow \tau + 1$  as well as  $\tau \rightarrow -\frac{1}{\tau}$ . In fact, under the fractional transformations  $SL(2, \mathbb{Z})$

$$\tau \rightarrow \frac{a\tau + b}{c\tau + d}, \quad (6)$$

which can be thought of as a discrete  $S$ -duality transformation, the field strength tensors transform as

$$\tau F_{\mu\nu}^+ \rightarrow (a\tau + b) F_{\mu\nu}^+, \quad \bar{\tau} F_{\mu\nu}^- \rightarrow (a\bar{\tau} + b) F_{\mu\nu}^-, \quad (7)$$

where  $a, b, c, d \in \mathbb{Z}$  with  $ad - bc = 1$  and a “bar” (over  $\tau$ ) denotes complex conjugation. The action associated with (5) is not invariant under (6)-(7), but the equations of motion are.

It should be emphasized that this symmetry in the equations of motion manifests only in the Yang-Mills sector and it will be gratifying if we can promote this to the fully interacting theory including fermions (even if at the level of equations of motion). We note that such  $S$ -duality symmetries appear naturally in supergravity as well as in string theories (more details can be found in the review articles [29]). In the present context we plan to generalize the discrete transformation in (6)-(7) to a continuous global  $S$ -duality transformation by introducing only one additional scalar field which is the  $S$ -duality partner of the axion. This partner scalar field couples to the scalar Yang-Mills Lagrangian density (much like the axion couples to the pseudoscalar density in (4)) in such a way that the theory is invariant under the continuous  $S$ -duality transformations. We call this  $S$ -duality partner (of axion), a scalar field  $\phi$ , with the *dilaton*. We emphasize here that we do not intend to invoke any string theoretic argument for the origin of this scalar field. In fact, a scalar field with a very light mass has also been introduced in the context of Jordan-Brans-Dicke theory of gravity [30, 31]. It is adequate, for the model we propose, that this should be a light, weakly interacting scalar field which is the  $S$ -duality partner of the axion, independent of any other identification. In fact, being the partner of the axion  $\chi$ , the  $\phi$  field is expected to share most of the attributes of the axion, it will be a weakly interacting, long lived and a very light mass particle. Since this global symmetry is not observed in nature, it will be broken (leaving possibly a discrete symmetry at the level of equations of motion) and we envision breaking this symmetry along the PCAC scenario much as in pion physics [32, 33].

We remind the reader that  $S$ -duality has been a very fertile domain of research in quantum field theory, supersymmetric theories, supergravity and string theories. There are very robust results, especially in theories with a large number of supersymmetries and in supergravity theories. Furthermore, in the context of string theory,  $S$ -duality has played a cardinal role in the investigation of nonperturbative properties of string theories. Moreover, a host of very important results in stringy black hole physics are derived by exploiting this symmetry. We believe that  $S$ -duality is a fundamental symmetry of nature

although it is broken. Our goal is more pragmatic and our approach is more phenomenological although we are clearly inspired by the powers of  $S$ -duality symmetry in various fields. We propose a model based on  $S$ -duality symmetry and hope that some of the consequences of our proposal can be tested in experiments at LHC as well as in cosmological experiments.

The plan of the paper is as follows. In section II, we describe the  $S$ -duality invariant Lagrangian densities for the axion-dilaton system as well as their coupling to fermions. In section III, we discuss the mechanism for the breaking of the  $S$ -duality symmetry. It is along the lines of PCAC in the study of pions. We derive the consequences following from such a scenario. In Section IV, we argue that the  $S$ -duality invariant coupling to fermions is universal for all fermion species including the neutrinos. This is in the spirit of string theory where the dilaton couples universally to all excitation of the string (its vev is the coupling constant of string theory) and axion, being its  $S$ -duality partner, couples universally as well. In this scenario neutrinos can acquire a small mass due to the spontaneous breaking of  $S$ -duality symmetry (independent of whether the neutrino is a Dirac or a Majorana particle).

## II. $S$ -DUALITY AND AXION-DILATON ACTIONS

In this section, we intend to present our model in some detail. We will follow a more efficient approach and construct  $S$ -duality invariant actions. We choose  $SL(2, \mathbb{R})$  as our  $S$ -duality group. (We note here that the group  $SL(2, \mathbb{R})$  is isomorphic to the symplectic group  $Sp(2, \mathbb{R})$  as well as to the generalized unitary group  $SU(1, 1)$ .) The Lie algebra  $\mathfrak{sl}(2)$  (which is isomorphic to the Lie algebra  $\mathfrak{su}(1, 1)$ ) is given by

$$[T_1, T_2] = -iT_3, \quad [T_2, T_3] = iT_1, \quad [T_3, T_1] = iT_2, \quad (8)$$

which differs from the  $\mathfrak{su}(2)$  Lie algebra in the sign of the first commutator and is connected with the fact that  $SL(2, \mathbb{R})$  is a noncompact group (unlike  $SU(2)$ ). As a result, it is not possible to have a finite dimensional unitary representation of  $SL(2, \mathbb{R})$  and we cannot choose all the finite dimensional generators  $T_1, T_2, T_3$  of the group to be Hermitian. For our purposes, it is sufficient to look at two dimensional representations of the group and a choice for the generators can be taken to be

$$T_1 = \frac{i}{2}\sigma_1 = \frac{1}{2}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad T_2 = -\frac{i}{2}\sigma_3 = \frac{1}{2}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}, \\ T_3 = \frac{1}{2}\sigma_2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad T_i^\dagger T_3 = T_3 T_i, \quad (9)$$

where  $i = 1, 2, 3$  and  $\sigma_1, \sigma_2, \sigma_3$  represent the three Pauli matrices. An alternative choice for the generators can be

taken to be

$$\bar{T}_1 = \frac{i}{2}\sigma_1 = \frac{1}{2}\begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \quad \bar{T}_2 = \frac{i}{2}\sigma_2 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \\ \bar{T}_3 = \frac{1}{2}\sigma_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad (10)$$

However, all such choices correspond to a change of basis and are related by a unitary transformation. For example, the choices of the generators in (9) and (10) are related by the unitary matrix ( $SS^\dagger = \mathbb{1} = S^\dagger S$ )

$$S = \frac{1}{\sqrt{2}}(\mathbb{1} - i\sigma_1) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}, \quad ST_i S^\dagger = \bar{T}_i, \quad (11)$$

with  $i = 1, 2, 3$ .

A  $2 \times 2$  matrix representing a general  $SL(2, \mathbb{R})$  transformation has the form (in a given basis)

$$\Omega = e^{-i\alpha_k T_k}, \quad (12)$$

where  $\alpha_k, k = 1, 2, 3$  denote the three real constant (global) parameters of the transformation. Note that the transformation matrix in (12) is real (since the generators are purely imaginary), namely,

$$\Omega^* = \Omega, \quad \Omega^\dagger = \Omega^T. \quad (13)$$

Since a finite dimensional representation of  $SL(2, \mathbb{R})$  is not unitary, the transformation matrices satisfy the condition (see (9))

$$\Omega^\dagger T_3 \Omega = \Omega^T T_3 \Omega = T_3, \quad \Omega^T T_3 = T_3 \Omega^{-1}, \quad (14)$$

so that  $(2T_3)$  can be thought of as the metric in the group space (note that  $(2T_3)^2 = \mathbb{1}$ ). Under a finite  $SL(2, \mathbb{R})$  transformation, a vector (in this case a doublet) transforms as

$$\Psi \rightarrow \Omega \Psi, \quad (15)$$

while a matrix in the adjoint representation transforms as

$$M \rightarrow \Omega M \Omega^T. \quad (16)$$

Let us choose the dilaton and the axion fields parameterizing the coset  $\frac{SL(2, \mathbb{R})}{U(1)}$  in the form

$$V = \begin{pmatrix} e^{-\phi} + \chi^2 e^\phi & \chi e^\phi \\ \chi e^\phi & e^\phi \end{pmatrix} = \begin{pmatrix} v_0 + v_2 & v_1 \\ v_1 & v_0 - v_2 \end{pmatrix} \\ = v_0 \mathbb{1} - 2iv_1 T_1 + 2iv_2 T_2, \quad (17)$$

which would transform under a  $SL(2, \mathbb{R})$  transformation in the adjoint representation as in (16)

$$V \rightarrow \Omega V \Omega^T. \quad (18)$$

Here we have identified

$$v_0 = \frac{1}{2}(e^{-\phi} + \chi^2 e^\phi + e^\phi), \\ v_1 = \chi e^\phi, \\ v_2 = \frac{1}{2}(e^{-\phi} + \chi^2 e^\phi - e^\phi). \quad (19)$$

It follows from (19) that

$$v_0^2 - v_1^2 - v_2^2 = 1, \quad (20)$$

which is a reflection of the condition

$$\det V = 1, \quad (21)$$

which holds for a special linear matrix. The matrix  $V$  in (17) is easily seen to satisfy (see (14))

$$V^T = V, \quad VT_3 = T_3V^{-1}. \quad (22)$$

We note here that the dilaton and the axion fields (as well as the matrix  $V$ ) introduced in (17) have zero canonical dimension. However, we can relate them to conventional spin zero fields with unit canonical dimension in  $3+1$  dimensions (see, for example, (4)) through the simple rescaling

$$\phi \rightarrow \frac{\phi}{f_s}, \quad \chi \rightarrow \frac{\chi}{f_s}, \quad (23)$$

where  $f_s$  denotes the scale of breaking for the  $S$ -duality symmetry (we will use this later).

As we have already mentioned in the introduction, the action describing the coupling of the dilaton and the axion to the Yang-Mills field cannot be written in a manifestly Lorentz and  $S$ -duality invariant manner although the equations of motion are. In this context following comments are in order. One can write a manifestly  $S$ -duality ( $SL(2, \mathbb{R})$ ) invariant Lagrangian density at the expense of manifest Lorentz invariance. Furthermore, it is necessary to introduce auxiliary gauge fields for this purpose. However, when these auxiliary fields are eliminated, the dynamical equations reduce to manifestly  $S$ -duality invariant and Lorentz invariant equations as noted in the introduction. We refer the interested reader to [29, 34] for more details on this.

The Lagrangian density for the dilaton and the axion can be written in the standard manner as

$$\begin{aligned} \mathcal{L}_{(\chi\phi)} &= -\frac{f_s^2}{4} \text{Tr} \partial_\mu V^{-1} \partial^\mu V \\ &= -\frac{f_s^2}{2} (\partial_\mu v_0 \partial^\mu v_0 - \partial_\mu v_1 \partial^\mu v_1 - \partial_\mu v_2 \partial^\mu v_2) \\ &= \frac{f_s^2}{2} (\partial_\mu \phi \partial^\mu \phi + e^{2\phi} \partial_\mu \chi \partial^\mu \chi), \end{aligned} \quad (24)$$

which is invariant under the  $SL(2, \mathbb{R})$  transformations (18). Note that under the scaling (23), the free dilaton part of the Lagrangian density in (24) takes the conventional form of a free spin zero boson theory. In terms of the complex moduli

$$\tau = \chi + ie^{-\phi}, \quad (25)$$

this Lagrangian density can also be written as

$$\mathcal{L}_{(\chi\phi)} = -\frac{f_s^2}{2} \frac{\partial_\mu \tau \partial^\mu \bar{\tau}}{(\tau - \bar{\tau})^2}, \quad (26)$$

where the moduli parameters (coupling constants) introduced in (5) can be related to the vacuum expectation value of the moduli defined in (25). The  $S$ -duality invariance of (24) leads to the Nöther current matrix

$$J_{(\chi\phi)}^\mu = -2iV^{-1} \partial^\mu V, \quad (27)$$

with the components given by

$$J_{(\chi\phi)i}^\mu = \text{Tr} T_i J_{(\chi\phi)}^\mu. \quad (28)$$

Explicitly the components take the form

$$J_{(\chi\phi)1}^\mu = v_0 \overleftrightarrow{\partial}^\mu v_1, \quad J_{(\chi\phi)2}^\mu = -v_0 \overleftrightarrow{\partial}^\mu v_2, \quad J_{(\chi\phi)3}^\mu = v_1 \overleftrightarrow{\partial}^\mu v_2. \quad (29)$$

To write the free fermion action in a  $S$ -duality invariant manner, let us consider a four component spinor  $\psi$  which may be a Dirac or a Majorana spinor. Let us next construct a doublet of  $SL(2, \mathbb{R})$  as

$$\Psi = \frac{1}{2\sqrt{2}} \begin{pmatrix} ((1 - \gamma_5) + i(1 + \gamma_5))\psi \\ ((1 + \gamma_5) + i(1 - \gamma_5))\psi \end{pmatrix}, \quad (30)$$

where  $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$  and  $\Psi$  transforms under  $SL(2, \mathbb{R})$  as a vector (15), namely,

$$\Psi \rightarrow \Omega \Psi. \quad (31)$$

The Dirac adjoint of this doublet (which transforms inversely under a Lorentz transformation) has the form

$$\bar{\Psi} = \frac{1}{2\sqrt{2}} (\bar{\psi}((1 + \gamma_5) - i(1 - \gamma_5)) \quad \bar{\psi}((1 - \gamma_5) - i(1 + \gamma_5))). \quad (32)$$

From the point of view of  $S$ -duality transformation, however, it is more useful to define an alternative adjoint of (30) which transforms inversely under both Lorentz as well as  $S$ -duality transformations as

$$\widetilde{\bar{\Psi}} = \bar{\Psi}(2T_3). \quad (33)$$

In fact, under a  $SL(2, \mathbb{R})$  transformation (31)

$$\widetilde{\bar{\Psi}} \rightarrow \bar{\Psi} \Omega^T (2T_3) = \bar{\Psi}(2T_3) \Omega^{-1} = \widetilde{\bar{\Psi}} \Omega^{-1}, \quad (34)$$

where we have used (14). With these we can now write the free fermion Lagrangian density as

$$\mathcal{L}_\psi = i\widetilde{\bar{\Psi}} \gamma_5 \not{\partial} \Psi = i\bar{\psi} \not{\partial} \psi, \quad (35)$$

which is easily seen using (31) and (34) to be manifestly  $SL(2, \mathbb{R})$  invariant. The Nöther current from the fermion sector can be derived to have the form

$$J_{(\psi)i}^\mu = \widetilde{\bar{\Psi}} T_i \gamma^\mu \Psi, \quad (36)$$

so that the total  $S$ -duality current is given by (see (28) and (36))

$$J_i^\mu = J_{(\chi\phi)i}^\mu + J_{(\psi)i}^\mu. \quad (37)$$

We can now introduce the interaction of the dilaton and the axion to the QCD fermions in a  $SL(2, \mathbb{R})$  invariant manner as follows. Here we have two possibilities. If we want the interaction to be of nongradient type, we can choose the Lagrangian density to be

$$\mathcal{L}_{1Y} = -ig_{1Y}\Lambda_{\text{QCD}}\tilde{\Psi}\gamma_5 V(2T_3)\Psi, \quad (38)$$

which is manifestly  $SL(2, \mathbb{R})$  invariant. Namely,

$$\begin{aligned} \mathcal{L}_{1Y} &\rightarrow -ig_{1Y}\Lambda_{\text{QCD}}\tilde{\Psi}\Omega^{-1}\gamma_5\Omega V\Omega^T(2T_3)\Omega\Psi \\ &= -ig_{1Y}\Lambda_{\text{QCD}}\tilde{\Psi}\gamma_5 V(2T_3)\Omega^{-1}\Omega\Psi = \mathcal{L}_{1Y}, \end{aligned} \quad (39)$$

where we have used (18), (31), (34) as well as (14). Here we have chosen the coupling constant  $g_{1Y}$  to be dimensionless and as a result the interaction strength involves a mass scale appropriate for the interaction and  $\Lambda_{\text{QCD}}$  denotes the QCD scale which is the appropriate scale for the interactions since we are considering only the QCD sector here. Explicitly, the interaction Lagrangian density has the form

$$\begin{aligned} \mathcal{L}_{1Y} &= -g_{1Y}\Lambda_{\text{QCD}}v_2\bar{\psi}\psi + ig_{1Y}\Lambda_{\text{QCD}}v_1\bar{\psi}\gamma_5\psi \\ &= -\frac{g_{1Y}\Lambda_{\text{QCD}}}{2}(e^{-\phi} + \chi^2 e^{\phi} - e^{\phi})\bar{\psi}\psi \\ &\quad + ig_{1Y}\Lambda_{\text{QCD}}\chi e^{\phi}\bar{\psi}\gamma_5\psi, \end{aligned} \quad (40)$$

which is manifestly parity conserving since the dilaton  $\phi$  is a scalar while the axion  $\chi$  is a pseudoscalar.

On the other hand, if we want the interaction to be of gradient type as in the case of the standard axion, we can consider the Lagrangian density

$$\mathcal{L}_{2Y} = -g_{2Y}\tilde{\Psi}(\partial V)(2T_3)\Psi \quad (41)$$

which can be shown to be manifestly  $SL(2, \mathbb{R})$  invariant following (39). Explicitly, the interaction Lagrangian density has the form

$$\begin{aligned} \mathcal{L}_{2Y} &= -g_{2Y}\bar{\psi}(\partial v_0)\psi, \\ &= -\frac{g_{2Y}}{2}(-e^{-\phi} + \chi^2 e^{\phi} + e^{\phi})\bar{\psi}(\partial\phi)\psi \\ &\quad - g_{2Y}\chi e^{\phi}\bar{\psi}(\partial\chi)\psi, \end{aligned} \quad (42)$$

which is manifestly parity conserving. It is worth noting here that even though this is a gradient coupling, the form of the interaction is quite distinct from that in the case of the standard axion (even after symmetry breaking as we will discuss in the next section). Furthermore, we point out here that, as in PCAC, the gradient coupling (41) can be generated from the free Lagrangian density for the fermion (35) by redefining the fermion fields as

$$\Psi \rightarrow e^{ig_{2Y}V(2T_3)}\Psi, \quad \tilde{\Psi} \rightarrow \tilde{\Psi}e^{-ig_{2Y}V(2T_3)}. \quad (43)$$

However, the nonderivative interaction (38) cannot be generated through a field redefinition, even from an invariant mass-like (nonderivative) term, since

$$\tilde{\Psi}\Psi = 0 = \tilde{\Psi}\gamma_5\Psi. \quad (44)$$

### III. SYMMETRY BREAKING

From the energy scales probed so far, there is no compelling experimental evidence in favor of an exact  $S$ -duality symmetry in nature. Therefore, this symmetry needs to be broken and if axion and dilaton are  $S$ -duality partners, their masses are expected to be of the same order of magnitude. Light weakly interacting particles (axions) are yet to be directly seen in laboratory and cosmological experiments, although there are stringent limits on their masses. For the scalar field  $\phi$  which is of stringy origin or corresponds to the Jordan-Brans-Dicke scalar, there are also constraints on its mass which are similar to that of the axion. We are encouraged by these observations. Furthermore, since the Lagrangian density for the axion and the dilaton (24) is defined on a coset much like the nonlinear sigma model for the pions (in the present context, the coset space is  $\frac{SL(2, \mathbb{R})}{U(1)}$ ), we consider the symmetry breaking to be completely parallel to the PCAC mechanism in the theory of pions. However, there would be important differences because unlike the theory of pions where the coset space  $\frac{SU(2) \times SU(2)}{SU(2)}$  is compact, here the coset space is noncompact. Furthermore, since we would like the axion field to have a nonzero vacuum expectation value (in order to have a nontrivial theta parameter in the theory), it is clear from the constraint (20) that  $v_0, v_1$  need to have nontrivial vacuum expectation value and (20) can in principle determine the vev for  $v_2$  to be nonzero as well.

To understand the vev structures better, let us note that (20) can be thought of as describing an (internal) anti-de Sitter space. Therefore, let us parameterize the space with hyperbolic coordinates as

$$v_0 = \cosh \Sigma, \quad v_1 = \sinh \Sigma \cos \xi, \quad v_2 = \sinh \Sigma \sin \xi, \quad (45)$$

where the constraint (20) is automatically satisfied. Comparing with (19) we can identify

$$\begin{aligned} e^{-\phi} &= \frac{1}{v_0 - v_2} = \frac{1}{\cosh \Sigma - \sinh \Sigma \sin \xi}, \\ \chi &= \frac{v_1}{v_0 - v_2} = \frac{\sinh \Sigma \cos \xi}{\cosh \Sigma - \sinh \Sigma \sin \xi}. \end{aligned} \quad (46)$$

It is clear from this that we can give vevs to both the dilaton as well as the axion consistent with (20) by requiring

$$\langle \Sigma \rangle = \Sigma_0, \quad \langle \xi \rangle = \xi_0, \quad (47)$$

where  $\Sigma_0, \xi_0$  are arbitrary constants and these would translate into the vevs

$$\begin{aligned} \langle e^{\phi} \rangle &= e^{\phi_0} = \cosh \Sigma_0 - \sinh \Sigma_0 \sin \xi_0, \\ \langle \chi \rangle &= \chi_0 = e^{-\phi_0} \sinh \Sigma_0 \cos \xi_0. \end{aligned} \quad (48)$$

From (45) we see that this can also be written as

$$\langle v_0 \rangle = \cosh \Sigma_0, \quad \langle v_1 \rangle = \sinh \Sigma_0 \cos \xi_0, \quad \langle v_2 \rangle = \sinh \Sigma_0 \sin \xi_0. \quad (49)$$

A direct consequence of the vevs is that the two different Yukawa interactions in (40) as well as (41) will generate trilinear interactions involving the fermions and the axion and the dilaton. In addition, (40) will generate masses for the QCD fermions upon shifting. Shifting (40) around the vevs (48) in a consistent manner with the scaling in (23), namely,

$$\phi \rightarrow \phi_0 + \frac{\phi}{f_s}, \quad \chi \rightarrow \chi_0 + \frac{\chi}{f_s}, \quad e^{-\phi} \rightarrow e^{-(\phi_0 + \frac{\phi}{f_s})}, \quad (50)$$

we obtain

$$\begin{aligned} \mathcal{L}_{1Y} = & -m_f \bar{\psi} \psi + im \bar{\psi} \gamma_5 \psi - \frac{g_Y \Lambda_{\text{QCD}}}{f_s} (\langle v_2 \rangle \phi + \langle v_1 \rangle \chi) \bar{\psi} \psi \\ & + \frac{ig_{1Y} \Lambda_{\text{QCD}}}{f_s} e^{\phi_0} (\chi + \chi_0 \phi) \bar{\psi} \gamma_5 \psi + \dots, \end{aligned} \quad (51)$$

where

$$m_f = g_{1Y} \Lambda_{\text{QCD}} \langle v_2 \rangle, \quad m = g_{1Y} \Lambda_{\text{QCD}} \langle v_1 \rangle. \quad (52)$$

The higher order terms in the interaction in (51) will be suppressed by inverse powers of  $f_s$ .

On the other hand, a shift of (42) leads to

$$\begin{aligned} \mathcal{L}_{2Y} = & -\frac{g_{2Y}}{2f_s} (-e^{-\phi_0} + \chi_0^2 e^{\phi_0} + e^{\phi_0}) \bar{\psi} (\not{\partial} \phi) \psi \\ & - \frac{g_{2Y}}{f_s} \chi_0 e^{\phi_0} \bar{\psi} (\not{\partial} \chi) \psi + \dots, \end{aligned} \quad (53)$$

We note here that the axion-nucleon-nucleon coupling constant has been estimated in the past [35] to be

$$g_{\chi NN} \sim 10^{-12}, \quad (54)$$

up to a model dependent multiplicative constant of the order of  $O(1)$ . In order to extract numbers from our model, we propose that the axion-quark-quark coupling constants in our model to satisfy

$$g_{1Y}, g_{2Y} \sim g_{\chi NN} \sim 10^{-12}, \quad (55)$$

up to a multiplicative constant so that the axion remains weakly interacting. With a trilinear coupling of this order of magnitude we note that the values of  $m_f, m$  in (52) will depend on the values of the parameters  $\Sigma_0, \xi_0$ . In particular, we note that since  $\chi_0 \sim \theta$  is expected to be small, it follows from (48) that  $\xi_0$  is constrained to be very close to  $\frac{\pi}{2}$  in which case (52) leads to

$$m \approx 0, \quad m_f \approx g_{1Y} \Lambda_{\text{QCD}} \sinh \Sigma_0 \approx 10^{-11} \text{ GeV}, \quad (56)$$

up to a constant (we have approximated  $\Lambda_{\text{QCD}} = 220 \text{ MeV} \approx 1 \text{ GeV}$ ). Here we have used the fact that with  $\xi_0 \approx \frac{\pi}{2}$ , it follows from (48) that

$$e^{\phi_0} \approx \cosh \Sigma_0 - \sinh \Sigma_0, \quad (57)$$

which leads to  $e^{-\Sigma_0} = e^{\phi_0}$  and if one were to relate  $e^{\phi_0} \sim g^2$  ( $g$  is the QCD coupling), then at the QCD scale,  $\Sigma_0$

has to be small. As a result, we conclude from (56) that the correction to the fermion masses in the QCD sector because of  $S$ -duality breaking is negligibly small.

It is worth noting here that the trilinear coupling in (53) involving the axion has a vector structure unlike the usual axial vector coupling of the axion. (As a result, there will be no  $\pi$ - $\chi$  mixing resulting from such an interaction.) Of course, this would have observational consequences. However, more than that the strength of this interaction is proportional to  $\theta$  (because of the factor of  $\chi_0 \sim \theta$ ) and correspondingly will be highly suppressed. In contrast, the dilaton trilinear interaction in (53) will be relatively more dominant which is experimentally interesting.

In this scenario, the dilaton and the axion are Goldstone bosons at this stage, just like the pions. As a result, their self interactions involve derivative terms and they remain massless even after shifting fields around the vevs. To introduce masses for the dilaton and the axion, we introduce a term of the form

$$\mathcal{L}_m = -\frac{f_s^2 m_s^2}{2} \frac{v_0}{v_0 - v_3} = -\frac{f_s^2 m_s^2}{2} (1 + e^{-2\phi} + \chi^2). \quad (58)$$

Shifting this around the vevs (48) (see (50)) we obtain

$$\begin{aligned} \mathcal{L}_m = & -\frac{f_s^2 m_s^2}{2} \left[ (1 + e^{-2\phi_0} + \chi_0^2) - \frac{2}{f_s} (e^{-2\phi_0} \phi + \chi_0 \chi) \right. \\ & \left. + \frac{1}{f_s^2} (2e^{-2\phi_0} \phi^2 + \chi^2) + \dots \right], \end{aligned} \quad (59)$$

which leads to masses for the axion and the dilaton of the forms

$$m_\chi^2 = m_s^2, \quad m_\phi^2 = 2e^{-2\phi_0} m_s^2 = 2e^{-2\phi_0} m_\chi^2. \quad (60)$$

If one were to identify  $e^{\phi_0} = g^2$ , the Yang-Mills coupling, then (60) would suggest that the mass of the dilaton would be relatively larger than that of the axion.

#### IV. SUMMARY AND DISCUSSION

In this work, we have proposed  $S$ -duality as a fundamental symmetry motivated by the issues in QCD which have led in the past to the introduction of the axion. We have identified the group  $SL(2, \mathbb{R})$  as the  $S$ -duality group and the “scalar” dilaton has been introduced as the  $S$ -duality partner of the pseudoscalar axion. The exponential of the vev of the dilaton is related to the QCD gauge coupling constant and we have introduced the interaction Lagrangian density for the fermions in the QCD sector in a  $S$ -duality invariant manner. However, in a more general setting such as the string theory, the vev of the dilaton is expected to control all coupling constants with the dilaton coupling to all matter. In this spirit, we propose universality of  $S$ -duality symmetry so that all fermions including quarks and leptons couple to the axion and dilaton in a  $S$ -duality symmetric manner as

given in (40) and (42). Of course, the coupling strengths and the interaction scale  $\Lambda$  can be different for different species. For leptons, for example, one can identify the interaction scale with the weak scale  $\Lambda = \Lambda_{\text{weak}}$  (unlike (38) where the relevant scale is  $\Lambda = \Lambda_{\text{QCD}}$ ).

Although our proposal is clearly influenced by string theoretic ideas, it is worth remarking here that our approach has been rather phenomenological so that the interactions are not derived from a (unique) fundamental theory. For example, as we have already pointed out, in string theory the vev of the dilaton field is expected to determine all the constants of the (low energy) theory (such as the fine structure constant, Yukawa couplings etc) in addition to the gravitational constant. However, we do not insist on any such requirement beyond the possible relation between  $e^{\phi_0}$  and the Yang-Mills coupling so that our goal is rather modest. String theory admits many more massless scalars and pseudoscalars when compactified to lower dimensions and there have been several studies on the phenomenological consequences of these moduli in string theory [36, 37]. There have also been attempts to establish a connection between the QCD axion and the string theoretic axion although the stringy axion models have failed to accommodate all the experimental bounds so far [19, 38]. Moreover, the implications of admitting ultra light axions of string theory in the cosmological context have been addressed recently [39]. In contrast, in our modest phenomenological approach we have only introduced one additional scalar, namely, the dilaton, to write Lagrangian densities in a  $S$ -duality invariant manner.

Our interpretation of the universality hypothesis is that there is a single Yukawa coupling for all the fermions belonging to the QCD sector (namely, for all quarks) and a single Yukawa coupling for all the leptons. We accommodate this proposition qualitatively as follows. As we have mentioned before, one can extract the axion-quark-quark coupling constant (which is of the same order of magnitude for the dilaton coupling) as in [35] (from the estimate of  $g_{\chi NN}$ ) as discussed in (55). Similarly, we can estimate the Yukawa coupling in the lepton sector from the estimate in [35]

$$g_{\chi ee} \sim 10^{-15}, \quad (61)$$

by requiring, as in (55), that  $(\tilde{g}_{1Y}, \tilde{g}_{2Y})$  are the analogs of  $g_{1Y}, g_{2Y}$  in the lepton sector)

$$\tilde{g}_{1Y}, \tilde{g}_{2Y} \sim g_{\chi ee} \sim 10^{-15}, \quad (62)$$

up to a multiplicative constant of order unity.

The universality hypothesis leads to some interesting consequences. As we have already pointed out, both the (trilinear) Yukawa interactions in (53) and (51) of the fermions with the dilaton and the axion are highly suppressed with our identification (55) which we expect to hold in the lepton sector as well (higher order interactions are suppressed further by inverse powers of  $f_s$ ). Moreover, since  $S$ -duality is broken, the fermions acquire a

mass, for example, as given in (52) (or the analogous formula in the lepton sector). The masses acquired by diverse families of fermions (due to  $S$ -duality breaking) would be extremely small since the relevant Yukawa couplings are small. We know that all the fermions (except the neutrino) acquire their masses in the standard model (electroweak theory) through their couplings to the Higgs and the corrections from the  $S$ -duality breaking in (52) to the masses of these fermions are negligible (see, for example, (56) for the QCD sector). However, since the neutrinos are massless in the standard model, the breaking of  $S$ -duality symmetry can provide a plausible alternative mechanism to generate a small neutrino mass. In fact, this mechanism can generate a mass for either Dirac or Majorana neutrinos and following (56) we can estimate this mass to be of the order of

$$m_\nu \sim \tilde{g}_{1Y} \Lambda_{\text{weak}} \sinh \Sigma_0 \approx 10^{-2} \text{eV}, \quad (63)$$

where we have used (62) as well as have approximated  $\Lambda_{\text{weak}} \sim 246 \text{GeV} \approx 1 \text{TeV}$  (keeping in mind that the relations (62)-(63) hold up to multiplicative constants of order unity). (Note that the corrections to the masses of all charged leptons will be of the same order of magnitude as (63), which is negligible, due to the universality hypothesis in the lepton sector.) In view of the experimental evidence for neutrino oscillations, the mechanism for generating a mass for the neutrino is an active area of study. If neutrino does acquire a mass through this mechanism, it may have interesting consequences since it would arise at the  $S$ -duality breaking scale  $f_s$  which lies between the GUT scale and the weak scale.

It is quite possible that axions and dilatons will be produced in LHC due to bremsstrahlung although the production cross sections will be quite suppressed compared to those for the Higgs production and the production of SUSY particles. However, these particles will travel long distances due to their weak coupling to matter. There is also a possibility that the axion and the dilaton produced at LHC will have a chance to get detected in underground and laboratory experiments which have been set up for dedicated searches of these particles. Moreover, although the dilaton proposed by us will have weak interactions, as weak as the axion, its interaction with matter will be much stronger than that of the stringy dilaton which is the light scalar partner of graviton in string theory and whose role in the cosmological domain has been a topic of considerable interest in the past [40–42]. The cosmological implications of 'our proposed' dilaton will also be of interest to examine carefully.

## Acknowledgments

We would like to thank Professors A. Melissinos and S. Okubo for valuable comments and discussions. We have benefited from numerous discussions with Costas Bachas, Pankaj Jain, Varun Sahni and Gabriele Veneziano. One

of us (A.D.) is supported in part by US DOE Grant number DE-FG 02-91ER40685. The other author (J.M.) is primarily supported by the People of the Republic of India and also partly through a Raja Ramanna Fellow-

ship by the Indo-French Center for the Promotion of Advanced Research: IFCPAR Project No. IFC/4104-2/2010/201.

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